Introduction

For a general state of stress at a point, there exists three mutually perpendicular planes at a point. The normal stress components on these mutually perpendicular planes are called principal stresses if shear stresses along this plane are zero. Since shear stress vanish on principal plane, the stress vector on principal plane is given by

While rotating the object in the three directions (, ) , at some location the shear stresses are getting vanished. That plane where shear stresses get vanish is called principal plane and the stresses are principal stresses.

Objective

To obtain the principal stresses of the stress at a point and to calculate the transformed stress components of the stress at a point when they are rotated about the three axes.

References

<http://en.wikipedia.org/wiki/Stress_(mechanics)>

<http://en.wikipedia.org/wiki/Rotation_matrix>

Theory

At every point in a stressed body there are at least three planes, called *principal planes*, with normal vectors **n**, called *principal directions*, where the corresponding stress vector is perpendicular to the plane, i.e., parallel or in the same direction as the normal vector **n**, and where there are no normal shear stresses. The three stresses normal to these principal planes are called *principal stresses*.

The components of the stress tensor depend on the orientation of the coordinate system at the point under consideration. However, the stress tensor itself is a physical quantity and as such, it is independent of the coordinate system chosen to represent it. There are certain invariants associated with every tensor which are also independent of the coordinate system. For example, a vector is a simple tensor of rank one. In three dimensions, it has three components. The value of these components will depend on the coordinate system chosen to represent the vector, but the length of the vector is a physical quantity (a scalar) and is independent of the coordinate system chosen to represent the vector. Similarly, every second rank tensor (such as the stress and the strain tensors) has three independent invariant quantities associated with it. One set of such invariants are the principal stresses of the stress tensor, which are just the eigenvalues of the stress tensor. Their direction vectors are the principal directions or eigenvectors.

A stress vector parellel to the normal vector **n** is given by:

Where λ is a constant of proportionality, and in this particular case corresponds to the magnitudes of the normal stress vectors or principal stresses.

Knowing that and , we have

This is a homogeneous system, i.e. equal to zero, of three linear equations where are the unknowns. To obtain a nontrivial (non-zero) solution for , the determinant matrix of the coefficients must be equal to zero, i.e. the system is singular. Thus,

Expanding the determinant leads to the *characteristic equation*

where

The characteristic equation has three real roots . The three roots are the principal stresses .

Let the the state of stress at appoint be

If the stress components is rotated about the three axes X, Y and Z at respectively, then the transformed stress matrix is given by

Where R = rotation matrix

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Example:

State of stress at a point

MPa

The stress components are rotated at

The stress invariants of the characteristic equation

are MPa

MPa

MPa

and principal stress are

MPa

MPa

MPa

The rotation matrix

and the transformed state of stress is

MPa